☑ Έστω $\varphi : \Sigma_3 \to GL(3,R)$ ο ομομορφισμος με τυπο

$$\varphi(1,2) = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} \quad \text{και} \quad \varphi(1,3) = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix}$$

Νδο φ μονομορφισμος

ΛΥΣΗ

$\Sigma_3 = \{e, f, f^2, g, fg, f^2 g\}$ και θδο $\ker\varphi = \{e\}$

Έστω $f = (1,2,3)$ και $g = (1,2)$

Τοτε $f^2 = (1,2,3)^2 = (1,3,2)$

$fg = (1,2,3)(1,2) = (1,3)$

$f^2 g = (1,3,2)(1,2) = (2,3)$

$\varphi(e) = I \in GL(3,R)$

$\varphi(f) = \varphi(1,2,3) = \varphi((1,3)(1,2)) \overset{ομομ.}{=\!=\!=} \varphi(1,3) \cdot \varphi(1,2) =$

$$= \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix} \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix} \in GL(3,R)$$

$\varphi(f^2) = \varphi(1,3,2) = \varphi((1,2,3)(1,2,3)) \overset{ομομ.}{=\!=\!=} \varphi(1,2,3)\varphi(1,2,3) =$

$$= \begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix} = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix} \in GL(3,R)$$

$$\varphi(fg) = \varphi(1,3) = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix}, \quad \varphi(g) = \varphi(1,2) = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} \in GL(3,R)$$

$\varphi(f^2 g) = \varphi(2,3) = \varphi((1,3,2) \cdot (1,2)) \overset{ομομ.}{=\!=\!=} \varphi(1,3,2)\varphi(1,2) =$

$$= \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix} \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} \in GL(3,R)$$

Αρα, $\varphi(\Sigma_3) = \left\{ I_3, \begin{pmatrix} 0&0&1 \\ 1&0&0 \\ 0&1&0 \end{pmatrix}, \begin{pmatrix} 0&0&1 \\ 0&1&0 \\ 1&0&0 \end{pmatrix}, \begin{pmatrix} 0&1&0 \\ 1&0&0 \\ 0&0&1 \end{pmatrix}, \begin{pmatrix} 1&0&0 \\ 0&0&1 \\ 0&1&0 \end{pmatrix} \right\}$

$\ker\varphi = \{a \in \Sigma_3 / \varphi(a) = I_3\} = \{e\} \Rightarrow \varphi$ μονομορφισμος